base-line C D is then measured with the greatest possible accuracy, and the angle formed at C and D in the directions *e* and *f* are obtained. Thus the two triangles C D *e* and C D *f* are completely given; for in each is a side (C D) and the two adjacent angles known. But thus also are their heights *e h* and *f h* given, and these added give the side *ef* in the great triangles. From *e* and *f* the two triangles formed towards A and B are measured, and this gives completely the two triangles A *f e* and B *f e*, as also their heights A *k* and B *k*, which added give the distance sought.[1] It will be seen at once that this method offers great advantages, especially if it be possible to obtain with the greatest accuracy the small base-line. This latter condition Bessel fulfilled at first to an astonishing degree, as he, by the introduction of a base-apparatus, attained the greatest accuracy. Bessel and Baeyer accomplished a degree-measurement between Memel and Trunz in 1831–36. They obtained for the mean latitude of the measured arc (54° 58′ 25″·5) a degree-length of 57142 toises. An operation was carried out by Maclear between 1836 and 1848 at the Cape of Good Hope, by which for south latitude 35° 43′ 20″ a degree-length of 56933 toises was obtained.

(*To be continued.*)

ON THE PRECESSION OF A VISCOUS SPHEROID[2]

I HAVE been engaged for some time past in the investigation of the precession of a viscous spheroid, with the intention of seeing whether it would throw any light on the history of the earth in the remote past. As some very curious results have appeared in the course of the work, I propose to give an account of part of them to the British Association.

The subject is, however, so complex and long, that no attempt will be made even to sketch the analytical methods employed.

In a paper of mine read before the Royal Society in May last, a theory was given of the bodily tides of viscous and imperfectly elastic spheroids; and this paper formed the foundation of the present investigation.

For convenience of diction I shall speak of the tidally disturbed body as the earth, and of the disturbing bodies as the moon and sun; moreover, in all the numerical applications, the necessary data were taken from these three bodies.

The effect of the internal friction called viscosity, is that the bodily tides in the earth lag, and are less in height, than they would be if the earth were formed of a perfect fluid.

An analytical investigation proved that the action of the sun and moon on the tides in the earth is such that the obliquity to the ecliptic, and the lengths of the day and month all become variable; the alteration in the length of the year remains, however, quite imperceptible.

But I will now explain, from general considerations, how the lagging of the tides produces the effects above referred to.

Let the figure represent the earth as seen from above the south pole, so that S is the pole, and the outer circle the equator. The rotation of the earth will then be in the direction of the curved arrow close to S. Within the larger circle is a smaller concentric one, one-half of which is drawn with a full line, and the other half with a dotted line. The full line semicircle is part of a small circle in S. latitude and the dotted one part of another small circle in the same latitude, but to the north of the equator. Generally, dotted lines indicate parts which are behind the plane of the paper.

It will make the explanation somewhat simpler, if we suppose the tides to be raised by a moon and antimoon diametrically opposite to one another; this, as is well known, is a justifiable modification of the true state of the case.

Then let M and M′ be the projections of the moon and antimoon on to the terrestrial sphere.

If the substance of the earth were a perfect fluid, or were perfectly elastic, the apices of the tidal spheroid would be at M and M′. If, however, there be internal friction, the tides will lag, and we may suppose the apices of the spheroid to be at T and T′. In order to make the subject more intelligible, the tidal protuberances are then supposed to be replaced by two equal heavy particles T and T′, which are instantaneously rigidly connected with the earth. This same idea was, I believe, made use of by Delaunay, in considering the ocean tidal friction.

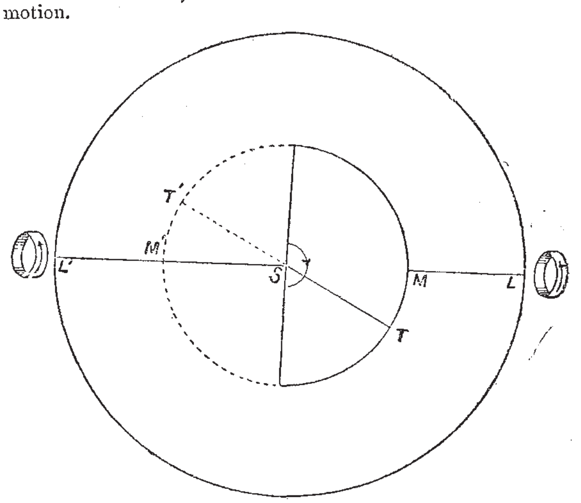
Then the attraction of the moon on T is greater than on T′; and that of the antimoon on T′ greater than on T. Hence, besides equal and opposite forces acting at the earth's centre, directly towards M and M′, there are small forces (varying as the square of the tide generating force) acting in the directions T M and T′ M′.

We will consider the effect on the obliquity first. These two forces, T M, T′M′, clearly cause a couple about the axis L L′ in the equator, which lies in the same meridian as the moon. The couple is indicated by the curved arrows at L and L′. Now, if the effects of this couple be compounded with the existing rotation of the earth, according to the principle of the gyroscope, it is clear that the south pole S tends to approach M and the north pole to approach M′. Hence supposing the moon to move in the ecliptic, the inclination of the earth's axis to the ecliptic diminishes; in other words, the obliquity of the ecliptic increases.

Next with regard to tidal friction; the forces T M and T′M′ produce a couple about the earth's axis, S, which tends to retard the earth's rotation.

Lastly, since action and reaction are equal and opposite, and since the moon and antimoon produce the forces T M, T′ M′ on the earth, therefore the earth must cause forces on the moon and antimoon in the directions M T and M′ T′. These forces are in the same direction as the moon's orbital motion; hence the moon's linear velocity is augmented. The consequence of this is that her distance from the earth is increased, and with that increase comes an increase of periodic time round the earth.

The consequences of the lagging of the earth-tides, therefore, are an increase of the obliquity to the ecliptic, a retardation of the earth's rotation, and a retardation of the moon's mean motion.



In this general explanation it is assumed that the lagging tides are exactly the same as though the earth were perfectly fluid, and as though the tide-*raising* moon were more advanced in her orbit than the true moon, whilst the moon which attracts the tidal protuberances was the true moon. That is to say, it is assumed that the tides raised are exactly the same as though the earth were a perfect fluid, save that the time of high tide is late, and that the tides are reduced in height.

Now although this serves in a general way to explain the phenomena which result from the supposition of the earth's viscosity, yet it is by no means an accurate representation of the state of the case.

In fact the internal friction sifts out the whole tide-wave into its harmonic constituents, and allows the different constituents to be very differently affected as regards height and phase.

Thus the lagging tide-wave is not exactly such as the general explanation supposes, and the nearer does the spheroid approach to absolute rigidity the greater does the discrepancy become.

The general explanation is a very fair representation for moderate viscosities, but for large ones it is so far from correct that the tendency for the obliquity to vary may become nil, and for yet larger ones the obliquity may tend to decrease.

[1] It is here assumed that *fe* is at right angles to C D, and A B at right angles to *fe*. There is no necessity for this condition, and it could never actually occur.

[2] A paper read at the Dublin Meeting of the British Association, by G. H. Darwin, M.A., Fellow of Trinity College, Cambridge.

A complete analysis of this state of things for various obliquities and viscosities shows that there is a great variety of positions of dynamical equilibrium, some of which are stable and some unstable.

Although there is all this variety with respect to the change of the obliquity, yet the tidal friction always tends one way, namely, to stop the earth's rotation.

It has already been remarked in the general explanation that the effect on the moon is a force tangential to her orbit accelerating her linear motion, and thus indirectly retarding her angular motion. But it appears that for a very great degree of stiffness and for large inclinations of the earth's axis to the ecliptic, this force on the moon may be actually reversed; so that the retardation of the moon's motion may actually be replaced by an acceleration.

To a terrestrial observer, however, unconscious of the slackening of the earth's diurnal rotation, it would be indifferent whether the moon were undergoing true retardation or true acceleration, for in every case there would result an apparent acceleration of the moon's mean motion.

It is obvious from what has been said that we have the means of connecting the heights and lagging of the bodily tides in the earth with an apparent secular acceleration of the moon's mean motion. I have applied these ideas to the supposition that the moon has an apparent secular acceleration of 4″ per century, and I find that if the earth were a homogeneous viscous spheroid, then the moon must be undergoing a secular *retardation* of 3″·6 per century, while the earth (considered as a clock) must be losing 14 seconds in the same time. Under these circumstances the effective rigidity of the earth must be so great that the bodily diurnal and semi-diurnal tides would be quite insensible; the *bodily* fortnightly tide would, however, be so considerable that the *oceanic* fortnightly tide would be reduced to one-seventh of its theoretical amount on a rigid nucleus, and the time of high water would be accelerated by three days.

The supposition that the earth is a nearly perfectly elastic body leads to very different results, which, however, I must now pass over.

From this and various other considerations, I arrive at the conclusion that the earth has a very great effective rigidity, and that the apparent acceleration of the moon's motion affords no datum for determining the amount of tidal friction on the earth.

Sir William Thomson has made some interesting remarks about the probable age of the earth in connection with tidal friction, and he derived his estimate of the rate at which the diurnal rotation is slackening principally from the secular acceleration of the moon. He fully admitted that his data did not admit of precise results, but if I am correct in the present conclusion, it certainly appears that his argument must lose part of its force.

The investigation of the secular changes, which such a system would undergo, is surrounded by great mathematical difficulties, but I think that I have succeeded in surmounting them by methods partly analytical and partly arithmetical.

In a communication of the present kind it would be out of place to consider the methods employed, and I will therefore only speak of some of the results.

There are two standards by which we may judge of the viscosity in the present problem—first the ordinary one, in which it is asserted that it requires so many pounds of tangential stress to the square inch to shear an inch cube through so much in such and such a time; and secondly, when the viscosity is judged of by the amount by which the behaviour of the spheroid departs from that of a perfectly fluid one; a numerical value for this sort of measure is afforded by the angle by which the crest of the tidal spheroid precedes the moon, when the obliquity to the ecliptic is zero.

Now it appears that if the earth possessed a viscosity which was not at all great as estimated by the tidal standard, yet the materials of the earth, when considered in comparison with the substances which we know, would be found to be a substance of very great stiffness—stiffer than lead, and perhaps nearly as stiff as iron. I see, therefore, no adequate reason why some part of the changes, which will be considered presently, should not have taken place during geological history.

The problem was solved numerically for a degree of viscosity, which would make the changes proceed with nearly a maximum rapidity. Estimated by the tidal standard, this is neither a very great nor a very small viscosity, for the crest of the semi-diurnal tide precedes the moon by 17° 30′.

I found, then, that if the changes in the system are tracked back for fifty-six million years, we find the day reduced to six hours fifty minutes, the obliquity to the ecliptic 9° less than at present, and the moon's period round the earth reduced to one day fourteen hours.

This very short period for the moon indicates of course that her distance from the earth is small. As the moon goes on approaching the earth the problem becomes much more complex, and, for periods more remote than fifty-six million years ago, I abandoned the attempt to obtain a scale of times. The solution up to this point shows that the times requisite for these causes to produce such startling effects are well within the time which physicists have admitted to have elapsed since the earth existed.

From this point in the solution the parallel changes of the obliquity, day and month, were traced without reference to time.

It appears, then (still looking backwards in time), that the obliquity will only continue to diminish a little more beyond the point already reached; for, when the sidereal month has become equal to twice the day, there is no longer any tendency for the obliquity to diminish, and for yet smaller values of the month the tendency is to increase again.

From this we learn that, when the day is equal to or greater than half the month, the position of the earth's axis at right angles to the plane of the moon's orbit is one of dynamical stability. The whole decrease of obliquity from the present value back to the critical point, where the month is equal to twice the day, is 10°. From this point in the solution back to the initial state to which the earth and moon are tending, the obliquity to the plane of the lunar orbit was neglected. I then found that the limiting condition, beyond which it was impossible to go, was one in which the earth and moon are rotating, fixed together as a rigid body, in five hours and forty minutes. This condition was also found to be one of dynamical instability, so that, if the month had been a little shorter than the day, the moon must have fallen into the earth, but if the month had been a little longer than the day the moon must have receded from the earth, and have gone through the series of changes, which were traced backwards up to this initial condition.

This periodic time of the moon of five hours forty minutes corresponds to an interval of only 6,000 miles between the moon's centre and the earth's surface. Moreover, if the earth had been treated as heterogeneous instead of homogeneous, this interval between the primeval earth and moon would have been yet further diminished, as also would be the common periodic time.

The conclusion, therefore, appears to me almost irresistible that if the moon and earth were ever molten viscous bodies, then they once formed parts of a common mass.

With respect to the obliquity of the ecliptic, the question is one of considerable difficulty, but, on the whole, I incline to the view that, while a large part of the obliquity may be probably referred to these causes, yet that there remains an outstanding part which is not so explicable.

Besides the results, of which the outlines have been given, I have obtained some others which, as I believe, will aid in the formation of a modified edition of the nebular hypothesis—such as some of the changes to which an annular satellite would be subjected.

One of the collateral results, which appeared in considering the secular changes of such a system as the earth, moon, and sun, was that a large amount of heat would have been generated in the interior of the earth by means of friction. If, then, it is permissible to suppose that any considerable part of these changes had taken place during geological history, Sir William Thomson's problem of the secular cooling of the earth would require some modification.

The magnitude of the undertaking has not allowed me time as yet to apply these ideas to the questions of the eccentricity and inclination of the orbit of the satellite, nor to the cases of other planets besides the earth.

I think, however, that I see in Asaph Hall's wonderful discovery of the Martian satellites, a confirmation of this theory. Their extreme minuteness has, I think, preserved them as a standing memorial of the primitive period of rotation of that planet. The Uranian system, on the other hand, appears, at least at first sight, a stumbling-block.

It is easy to discern in the planetary system many *veræ causæ* which tend to change its configuration, but it is in general very hard to give any quantitative estimate of their effects.

It will have been seen that in the investigation of which I have given an imperfect account, free scope has been given to speculation, but that speculation has been governed and directed in every case by appeal to the numerical results of a dynamical problem, and I therefore submit that it stands on a different footing from the numerous general speculations to which the nebular hypothesis has given rise.

NATURAL SCIENCE IN HUNGARY IN THE LAST TEN YEARS

FEW of the readers of NATURE are aware that Hungary has of late years become the scene of active efforts in science, and especially the natural sciences.

The following sketch of an article, written by Mr. Coloman Szily, member of the Academy of Sciences, and Professor of Physics at the Polytechnic of Budapest, and published in the *Budapesti Szemli* (*Budapest Review*), may therefore not be altogether without interest.

The first active sign of native scientific life in this direction in Hungary was the founding of the Academy of Sciences in 1830. Up to that time there were single men of science, but no organised scientific life. But the chief object aimed at by the Academy was the cultivation of the national language, and the excessive zeal with which it pursued this aim did much harm to the cause of the natural sciences here. An erroneous attempt to substitute purely Hungarian words for the mathematical and other scientific expressions universally accepted elsewhere, threw great obstacles in the way of the progress of the natural sciences in our country.

This and other errors soon brought a reaction.

The "General Assembly of Physicians and Naturalists" was soon started amid general enthusiasm. The meetings of the assembly were held yearly in different cities from 1841 up to 1848, and then renewed in 1863, after a cessation of fifteen years, caused by political events. A yearly report was issued, containing the various papers read at the meeting, as well as an account of physical characteristics of the district in which it was held.

Far more brilliant was the success reached in the cultivation and promulgation of the natural sciences by the "Természettudományi Társulat" (Society of Natural Sciences), which was started at the same time. By 1848 the number of the members rose to more than 400. Its first yearly report was then issued, and a contract made for the starting of a scientific magazine, entitled "Magyar Iris." This powerful start, which was made independently of the Academy, and which proved of ever-increasing importance, could not remain without effect. In 1844 a proposition that the two classes of mathematicians and naturalists might hold their meetings and carry on their financial and other operations separately from the rest was partially accepted. Some years later (1861) it only required a single lecture (of Prof. Joseph Izabo's) to bring the whole Academy to pronounce a resolution against the attempt to Magyarise the nomenclature.

In the meantime the events of 1848-49 were followed by a long period of despotism, which tended to paralyze all attempts at association. The most distinguished men of science were forced off the field of action, the Academy could hold no meetings, the Society of Natural Sciences was on the brink of dissolution, its members were scattered, its collections had to be given away for lack of funds to pay the rent of the accommodation needed for them, and it was barely able within ten years to publish two of its yearly reports.

Between 1850 and 1860 the nation began to breathe more freely. Its very first efforts were turned towards the advancement of science. A very fine building was raised for the Academy, and its capital considerably increased, by means of private subscriptions. It thus became able to do much more than it did previously, both for the improvement of our native language and for the cultivation of the various branches of science. In 1860 it appointed a committee of mathematicians and naturalists, whose duty it was to explore the whole country and give an account of its natural and technical features. Ever since the year 1868 Government has devoted the yearly sum of 5,000 florins to the furtherance of the labours of this committee. But this sum frequently proves insignificant. Fourteen volumes of the publications of the committee, entitled, "Scientific Treatises Relating to Home Topics" (Természettudományi közleménysk, vonatkozólag a hazai viszonyokra), edited by its secretary, Prof. Joseph Szabo, have already appeared. At the same time with this the Academy started a second series of periodicals for publishing mathematical and scientific treatises, not confined to topics within the limits of our country. They appeared yearly from 1860 to 1867, six volumes in all.

After the renewal of constitutional life in 1867 our naturalists were also filled with a strikingly-increased zeal for labour. The Academy has up to the present day issued thirty-two volumes in all. Many articles, treating of the original researches of our naturalists, have appeared in foreign periodicals. The meetings of the department of natural sciences in the Academy have of late borne witness to a truly diligent and scientific spirit, there scarcely being one in which less than six or eight treatises have been presented upon topics of original research.

In 1868, though no preliminary agreement had taken place between the two institutions, the department of Naturalists of the Academy, having arrived at the conviction that the popularising of the natural sciences was not their calling, abandoned the attempt, and decided that they should henceforth direct their efforts solely to the cultivation of the sciences and the making of scientific researches in our country, while the Society of Natural Sciences took upon itself the spreading and popularising of them. To this end the Society started a monthly periodical; the number of members of the Society rising in the very first year of its existence from 600 to 1,600, the second year to 2,200, until at present it borders upon 4,800. A short time later the Society began to arrange lectures connected with experiments for the benefit of the public. These lectures have now been kept up for eight years, and the large lecture-hall in which they have been held has always been crowded with hearers. As long as it was possible these lectures were published in the *Journal of Natural Sciences*; now, however, they appear in the form of a new series of publications under the title, "Collection of Popular Treatises upon Topics pertaining to the Natural Sciences." In 1872 the Society again started a new undertaking, namely, the translating into Hungarian and issuing of foreign works of a popular kind upon the natural sciences. The result of this undertaking, which has enjoyed the support of 1,500 subscribers, as well as a yearly aid from the Academy, has up to the present time been the issuing of twelve volumes, such as "Geologie der Gegenwart," by von Cotta; Darwin's "Origin of Species," Helmholtz's "Populäre Vorlesungen," Huxley's "Lessons in Physiology," Lubbock's "Prehistoric Times," Proctor's "Other Worlds than Ours," and Tyndall's "Heat as Motion." An Anthology has also been compiled, containing a treatise from every scientific author who has contributed to the popularising of the natural sciences, from the time of Arago and Humboldt downwards, and a volume containing the complete works of the late Julius Greguss. These books always find a large number of purchasers.

But there is another branch of activity which is of more importance, perhaps, than all those, namely, its efforts for the encouragement of original research. From 1870 the legislature of our country, in appreciation of the labours of the Society, has voted a yearly sum at first of 5,000, and afterwards of 4,000 florins, for the promotion of such researches as stand most nearly connected with the interest of our country, and the publication of an account of the same. In this series of publications the following have appeared up to the present time:—"The Rise and Fall of Tide in Fiume Bay," by Emile Stahlberger; "The Ice Grotto of Dobsina," by Dr. Joseph Alexander Krenner; "Sketch of the Ligaridas of Hungary," by Dr. Géza Horváth; "The Spiders of Hungary: Vol. I. General Part," by Otto Hermann; "The Iron Ores and Iron Products of Hungary, with Special Reference to the Principal Chemical and Physical Qualities of the Iron," by Anton Kerpsly.

In searching out and making known the physical characteristics of our country, the Society of Geologists (Magyar földtani társulat), founded in 1851, can also boast considerable merits. The Society also publishes a monthly periodical under the title of *Geological Review*.

In 1872 there was also a Geographical Society founded, which did not aim so much at the advancement of geographical researches as at keeping the public informed of any progress made on this field by means of a two-monthly review.

This active interest in the natural sciences is not confined only to our capital, but has taken root throughout the country. To prove this we have but to note the interest manifested in the labours of the Society of Natural Sciences in all grades of